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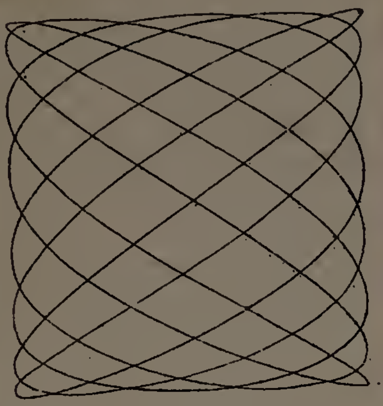
DESCRIPTION
OF A
MACHINE FOR DRAWING
THE
CURVES OF LISSAJOUS.

BY PROF. EDWARD C. PICKERING,

Institute of Technology, Boston, Mass.

From the Journal of the Franklin Institute.



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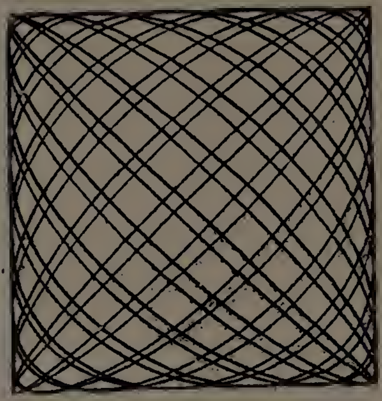

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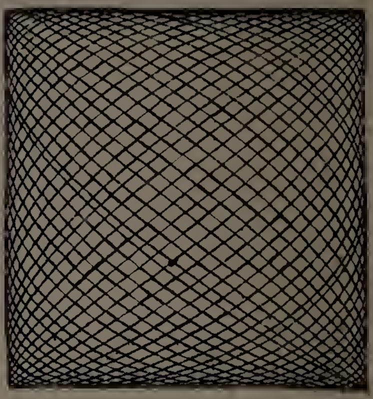

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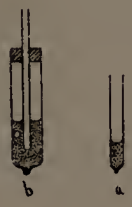

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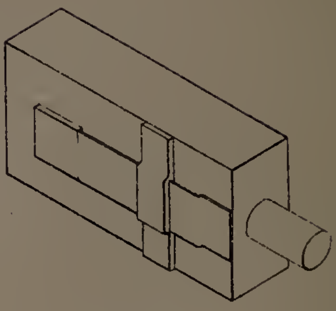

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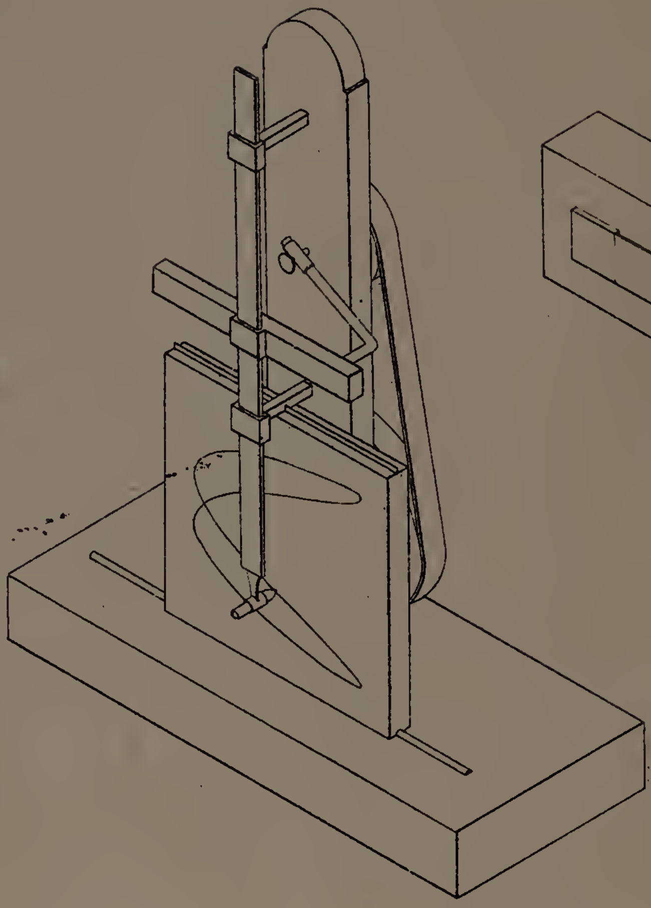
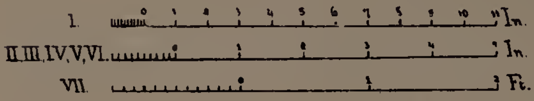


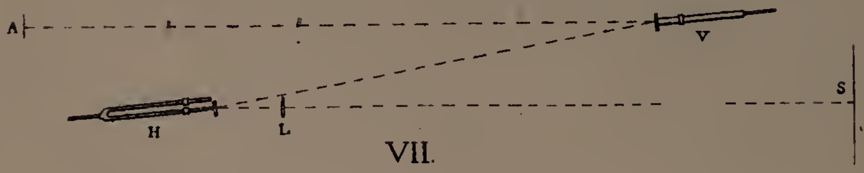
I

A Machine for Drawing

LISSAJOUS' VIBRATIONS.

BY E C PICKERING





VII.

Charles S. Minot M.D.
Nov. 30, 1892.

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DESCRIPTION
OF A
MACHINE FOR DRAWING
THE
CURVES OF LISSAJOUS.

BY PROF. EDWARD C. PICKERING,

Institute of Technology, Boston, Mass.

From the Journal of the Franklin Institute.

In 1857, M. Lissajous studied the curves produced in the following experiment. Mirrors are attached to the prongs of two tuning forks, whose planes of vibration are at right angles to one another. A ray of light falling on the first, is reflected from it to the second, and is then projected on a screen by a lens. If, now, the fork which vibrates in a horizontal plane is sounded, the motion thus imparted to its mirror causes the luminous point on the screen to describe a horizontal line. Sounding the other fork in the same way, produces a vertical line. When both vibrate at the same time, various curves are produced, dependent on the relative pitch of the two forks.

When showing this experiment to an audience, it is desirable to have a set of curves, drawn on a large scale, for comparison. As, however, their geometrical construction is somewhat laborious, I have devised a machine, represented in the adjoining photo-lithograph, by which they may be drawn mechanically.

The paper on which the curves are drawn receives a horizontal motion to and fro, while, at the same time, the pen is moving vertically up and down. These motions are imparted thus:—Two wheels (Fig. I.), connected by a belt, carry cranks, of which the lower one moves in a vertical slit in the rear of the square board to which the paper is attached. This board is kept in place below by a rod, over which it rolls, and above by a guide of sheet-brass. If, now, the lower wheel is turned, the paper moves horizontally through a distance equal to twice the crank-arm. A similar motion is imparted to the pen by a horizontal slit, in which the upper crank slides, and by which the pen is moved up and down. The rod is made flat to prevent its turning: or, it may be made circular, if we fix its slit in guides.

As a belt connects the two wheels, turning one moves both pen and paper. All the different phases of the curves corresponding to any given ratio of the number of vibrations of the two forks, may be drawn by merely sliding the belt over one of the wheels. As the ratio of the diameter of the latter corresponds to that of the number of vibrations of the forks, a series of wheels of different sizes are made, which, when combined two-and-two, give an almost endless variety of curves. Perhaps the best ratios, if we have four wheels, are $1 : 1\frac{1}{2} : 1\frac{3}{5} : 2$, by which we get curves corresponding to $\frac{16}{15}$, $\frac{5}{4}$, $\frac{4}{3}$, $\frac{3}{2}$, 2 and $\frac{8}{5}$, or to the semitone, major third, fourth, fifth, octave and minor sixth.

It may be proved that the curves coincide with those of Lissajous, since the angles described by the cranks are proportional to the diameter of the wheels; also, the distances of either pen or paper from the middle points of their paths are proportional to the sines of these angles. In other words, calling v the angle traversed by the upper crank, a, the ratio of the two wheels, and B, an angle dependent on the phase of the vibration, we have $y = \sin v$, and $x = (av + B)$ equations defining Lissajous' curves.

To obtain curves corresponding to every ratio of the forks, expanding wheels may be used, in which, by turning a spiral guide, six sectors are made to approach or recede from the centre. With two such wheels, whose diameter can be altered from 3 to $4\frac{1}{4}$ inches, and from $4\frac{1}{4}$ to 6 inches, all the curves within the limits of an octave can be drawn. By a set screw the crank-arms may be shortened so that smaller curves may be drawn, also those inscribed in a rectangle instead of a square.

To draw the curves in ink, it was necessary to devise some new kind of pen. That represented in Fig. V*a*, answered as well as could be desired. It is easily made from a glass tube, by thickening and nearly closing one end with the blowpipe, placing a drop of water in it, and grinding it flat on a file. When filled, the convex liquid surface at the small end retains the ink, while, when pressed on the paper, the flow takes place with great smoothness. Fig. V*b*, shows a modification which is useful for drawing diagrams. The pen is made so as to contain a large supply of ink, which is kept from running out by closing one end by a cork, through which passes a tube, as in the figure. The pressure is then constant, and equals that of a column whose height is the distance from the point of the pen to the end of the inner tube. By varying this distance, we can use a high pressure for thick ink, and a low pressure for that which is more fluid.

In Figs. II., III. and IV., are given specimens of curves drawn by the apparatus above described, to show its practical working. In Fig. II., the ratio is 5 to 8, or the curve of the minor sixth; in Fig. III. it is 15 to 16, or a semitone; and in Fig. IV. about 3 : 4, more nearly 44 : 59, or a sharp fourth. In the latter case the curve does not close exactly, since the ratio is incommensurable. As these curves are here reproduced by photography, the slightest errors are visible, and they furnish a good test of the workmanship of the machine. Probably still greater accuracy would have been attained had the apparatus by which they were drawn been constructed by a professional instrument-maker.

When projecting the curves of Lissajous it is somewhat difficult to show the more complicated forms, owing to the large size of the luminous point compared with the curves themselves. I find the arrangement of Fig. VII. gives the best results. A is an aperture through which a ray of sunlight enters; V and H are the two forks, one vibrating in a vertical, the other in a horizontal plane; L is a lens of 6 feet focus, by which the curve is projected on the screen, S. The size of the curve depends only on the distance, L S, or on H S, if the light traverses the lens before meeting the mirror, H, and, therefore, the lens must be brought near the aperature, A. But, if we use a lens of short focal length, the luminous point is greatly magnified, since its diameter is proportional to $\frac{\mathrm{L\,S}}{\mathrm{A\,V} + \mathrm{V\,H} + \mathrm{H\,S}.}$ I therefore use a lens of 6 feet focus, and remove V to a distance of $3\frac{1}{2}$ feet from H, so that A V + V H + H L and L S shall be conjugate foci. This would be the case in Fig. VII., when L S is 18 feet, or in a room 20 feet wide. It is not represented in its true size, in the figure, for want of space.

Lenses suitable for these projections of any size not exceeding 8 inches in diameter, and 6 feet focus, may be obtained at a low price from the makers of cosmoramas. Although not achromatic, their aberration is exceedingly small, from their great focal length. They are useful for many purposes, particularly in large lecture-rooms; thus, in projecting the solar spectrum, I find them preferable to much more expensive achromatic lenses of short focal length. Again, by using one as an object-glass, a very good telescope for class purposes may be made for a few dollars, capable of showing all the more familiar astronomical objects, such as satellites of Jupiter, ring of Saturn, prominent nebulæ, &c., and which would, I think, be of great use to many institutions where a more expensive instrument could not be afforded.

Fig. VII. represents a substitute for the tuning forks in the experiment of Lissajous. It consists of a rectangular box, in which is an aperture just filled by a square plate, carrying the mirror. This is attached to an elastic strip of metal, which vibrates when air is forced into the box, like the tongue of a melodeon-reed. To vary the rapidity of vibration, the metal strip or tongue passes through a guide, by which means its length may be altered, and this may be done with the utmost precision by a screw not represented in the figure. This adjustment is not necessary in the reed replacing the second tuning fork, since a range of more than an octave is readily obtained by the method here described. The advantages claimed for this apparatus over the tuning forks are, that the angular amplitude is much greater, hence the curves on the screen are of very large size; that they may be maintained indefinitely by keeping up the supply by the bellows, and most important of all, by merely turning the screw, the progressive changes of one curve into another may be shown, so that, in a few moments, all possible combinations may be exhibited.

My experiments with this apparatus are not yet completed, as I find that the vibrating part must be made with a good deal of care, or the curve will not be perfect. In fact, in a preliminary experiment, in which I used too feeble a spring, I found that it answered admirably for showing the difference of quality of different sounds. By touching the mirror with the point of a knife, so as to produce a rattling noise, I obtained, with a single reed, instead of a right line, a complicated curve, which varied with every change in the sound.